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Time Bunching of Slow Positrons for Annihilation Lifetime and Pulsed Laser Photon Absorption Experiments

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Abstract. Methods are described for gathering positrons from an extended region and causing them to impact upon a target surface within a small time interval. The most optimistic of these proposed schemes suggests that one should be able to produce $\sim 10^{-9}$ s positron pulses with peak intensities of $\sim 10^{11}\,s^{-1}$ starting with a ~ 200 m Ci $^{58}Co\ \beta^+$ source. These pulses should be useful for studying time-dependent interactions of positrons at surfaces.

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Slow positron beams are being used in a number of laboratories to study the interaction of positrons with clean surfaces [1–5] and in particular to study the free positronium which is formed copiously when positrons impinge upon a solid surface in vacuum [6]. It is often necessary or desirable to know accurately the time of arrival of the positrons at the target. For instance, the measurement of the lifetime of triplet positronium requires that its time of formation be known with an uncertainty small compared to its 142 ns mean life. Experiments designed to photo-induce transitions between various positronium levels (most notably the first order Doppler free $1S \rightarrow 2S\,2\gamma$ transition [7]) or positron surface state levels require a large flux of light from a necessarily low duty cycle pulsed laser. In this case the measurements are only possible if we know that a positron is at the target during a reasonably large number of the laser pulses which typically have ~ 10 ns durations.

One way of telling when a positron has struck a target is to detect the secondary electrons [8] which leave the surface after the positron impact. An MgO coated channeltron used both as the target surface and as the secondary electron detector has been the basis for recent measurements of the lifetime of triplet positronium in vacuum [9–11]. The lack of versatility associated with using a channeltron as the positron target could easily be overcome by detecting the secondary electrons from a separate target which could thus have different surface properties from the channeltron [12]. A more exotic timing method, but one which is of much lower efficiency, is to form positronium in its first excited state [13] and detect the Lyman-α photon that signals the transition to the ground state [14, 15].

A somewhat different approach would be to manipulate the positron beam so that particles can only arrive at preselected times. In the simplest realization of this scheme one chops the positron beam by a voltage pulse on the target or on a grid element. Since one usually needs a rather long period between pulses, this method results in a low probability of there being a positron in the pulse interval. However, in many experiments for which timing information would be useful, the only requirement on the energy of the incident positrons is that it be small enough ($E \ll 1$ keV) that the positrons are implanted into the target surface at a depth small compared with their diffusion length. In this way most of the implanted positrons can diffuse back to the surface in a time much shorter than their annihilation lifetime in the solid material of the target. Having diffused back to the surface, the positrons form positronium [2] or become trapped in their image potential well at the surface [5], etc. In these experiments with only crude requirements on the energy spread of the incident positrons, we can sacrifice the very good initial energy definition of the positron beam [1, 16] ($\Delta E \approx 0.25$ eV) to create a swarm of positrons which will arrive simultaneously at the target.

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The basic constraint on any attempt to bunch particles is Liouville's theorem which states that the volume in phase space occupied by an ensemble of particles is conserved in the absence of dissipative forces. Let us estimate what can be achieved by distorting the phase space volume occupied by a beam of positrons. Suppose the beam is produced by a source of S positrons per second with an initial velocity spread $\sim \pm v_0$ in each direction in 3D. The initial linear density of the beam is $\sigma_0 \approx S/v_0$. Now by an appropriate acceleration of each velocity component $v_i \to A v_i$ we may increase the linear density to $\sigma = A^3 \sigma_0$ keeping the transverse beam dimensions fixed. The instantaneous rate of positrons impacting the target can then be as high as $\Gamma = \sigma v \approx A^4 S$. It is evident that with $A = 20$ corresponding to final state energy spreads of ~ 100 eV one can in principle obtain an instantaneous $\sim 10^5$-fold increase in the effective source strength.

Three ways of effecting a time bunching of a positron beam will now be described. In the first one applies a time dependent accelerating voltage $V(t)$ to the beam. If we choose $V(t) = ml^2 t^{-2}/2$ the positrons will arrive simultaneously at $t=0$ at a distance l from the accelerator. Since we are only manipulating velocities along the beam axis, the gain in instantaneous positron flux is only A^2 which in this case is the ratio of the maximum accelerating voltage V_m to the initial spread in beam energy ε, $A^2 = V_m/\varepsilon \approx 400$ typically. This technique has been used successfully to measure the time of flight spectrum for positronium formed at a surface [17]. The second bunching method has theoretically the same effectiveness as the first but should be easier to implement. One suddenly applies a potential which varies quadratically with distance from the target, $V = kz^2/2$. All the positrons in this potential arrive simultaneously at $z=0$ if they initially have very low velocities since they are in a simple harmonic oscillator potential. Again, the gain in beam flux is $\sim A^2$ or the ratio of the maximum value of V to the initial energy ε when $V=0$.

In the third bunching method, we first trap a large number of positrons in a magnetic bottle by giving them a high transverse energy E after they get into the bottle. The small longitudinal energy ε is unaffected by this. Having collected a sufficient number of positrons, the quadratic potential $V = kz^2/2$ is suddenly turned on to collect them all simultaneously at the target. The flux gain is now $\sim A^4 = (E/\varepsilon)\cdot(V_m/\varepsilon) \approx 10^5$.

Considering now the first bunching method in detail, suppose at time t' a swarm of positrons is emitted at $z=0$ with phase space density f given by

$$f(x, V_z, t') = \frac{mv_z}{w}\,\theta(\tfrac{1}{2}mv_z^2 - V(t'))\cdot\exp\left[\frac{V(t') - \frac{1}{2}mv_z^2}{w}\right]\delta(z)\,, \tag{1}$$

where $\theta(x)$ is the unit step function. For convenience we are assuming a Boltzmann velocity distribution corresponding to the positrons having a mean longitudinal energy w. At a later time t the swarm has become

$$f(z, v_z, t', t) = \frac{mv_z}{w}\,\theta(\tfrac{1}{2}mv_z^2 - V(t'))\cdot\exp\left[\frac{V(t') - \frac{1}{2}mv_z^2}{w}\right]\delta(z - v_z(t-t'))\,. \tag{2}$$

Since $\partial f/\partial z = v_z^{-1}\partial f/\partial t$, $f(z, v_z, t', t)$ is solution of Boltzmann's equation, and we have normalized f so that

$$\iint f(z, v_z, t', t)\,dz\,dv_z = 1\,. \tag{3}$$

The total phase space density at time t is the sum of all contributions from times $t' < t$:

$$\begin{aligned} g(z, v_z, t) &= S\int_{-\infty}^{t} f(z, v_z, t', t)\,dt' \\ &= \frac{Sm}{w}\,\theta(\tfrac{1}{2}mv_z^2 - V(t - z/v_z))\cdot\exp\left[\frac{V(t - z/v_z) - \frac{1}{2}mv_z^2}{w}\right], \end{aligned} \tag{4}$$

where S is the source strength in positrons per second. We want to know the flux of particles at $z = z_0, t = 0$:

$$\begin{aligned} \phi(z_0, 0) &= \int_0^{z_0/t_0} v_z g(z, v_z, 0)\,dv_z \\ &= \frac{S}{w}\int_0^{V_m} \theta\left(u - V\left[\frac{-z_0}{v_z}\right]\right)\exp\left[\frac{V(-z_0/v_z) - u}{w}\right]du\,, \end{aligned} \tag{5}$$

where $u = mv_z^2/2 \leqq V_m$. It is clear that ϕ is greatest if $V\left(\frac{-z_0}{v_z}\right) = u$ or $V(-t) = mz_0^2 t^{-2}/2$, in which case $\phi(z_0, 0) = SV_m/w$ as expected. V_m is the maximum value of $V(t)$ which occurs at $-t = t_0$ and we assume $V(t) = 0$ for $t > -t_0$. For a general time t we find approximately

$$\phi(z_0, t) \approx \frac{Sm}{w}\,\theta(-t)\int_0^{z_0/t_0} \exp\left(\frac{mv_z^3 t}{wz_0}\right)v_z\,dv_z\,. \tag{6}$$

This expression yields the following series expansion:

$$\phi(z_0, t) = \theta(-t)\frac{SV_m}{w}\sum_{k=0}^{\infty}\frac{2}{k!(3k+2)}\left(\frac{2V_m t}{wt_0}\right)^k \tag{7}$$

and for large $-t$ becomes

$$\phi(z_0, t) = \theta(-t)\frac{SV_m}{w}\left(\frac{2V_m t}{wt_0}\right)^{-2/3}(2/3)\Gamma(2/3)\,. \tag{8}$$

This function is plotted in Fig. 1. The unit of time is $\tau = wt_0/2V_m$ and the total number of positrons in the pulse is $N_0 \approx 3.30 S t_0$. In the experiment of [17], $z_0 = 200$ cm, $t_0 = 320$ ns, $V_m \approx 80\,V$, $\tau \approx 0.5$ ns and

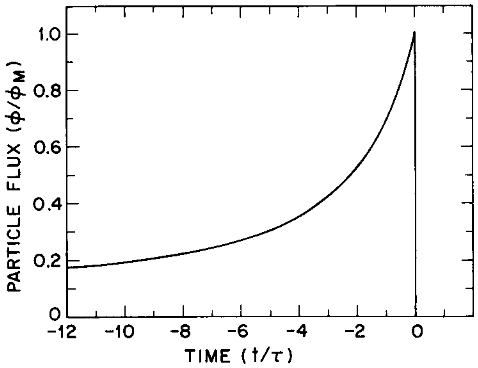

Fig. 1. Ideal pulse shape of a positron beam accelerated by a $1/t^2$ voltage pulse. The unit of time is $\tau = wt_0/2V_m$ where w is the mean energy spread of the unaccelerated beam (assumed to have an exponential energy distribution) and t_0 is the time prior to $t=0$ at which the acceleration pulse reaches its maximum value V_m. The peak flux is ϕ_m

$N_0 = S \times 1\,\mu s$. In the actual experiment, the positron pulse was clipped short by a <10 ns pulse on the target and the actual number of positrons per pulse was roughly $N_0/4$.

The $1/t^2$ positron pulse has a long tail, as shown in Fig. 1, which is made even less precise by the difficulty of shaping the required voltage pulse. This suggests that we might do better by only pulsing the target. In this second scheme, the incident positron energy is made as low as possible, say $\varepsilon \approx 0.25$ eV, to make a high linear density $\sigma = 2S/v$ with $v = 3 \times 10^7\,\mathrm{cm\,s^{-1}}$. By a series of accelerator rings (see Fig. 2) we suddenly apply a potential $V(z,t) = kz^2\theta(t)/2$ to the positrons in a segment l of the beam. In this way we collect $N_0 = 2Sl/v$ positrons in a pulse whose maximum flux is SV_m/ε, where V_m is the maximum value of $V(z,t)$. If we eliminate straggling positrons by collecting only those positrons in the segment $l<z<2l$, the time spread of the pulse arriving at $z=0$ is $\Delta t \approx vml/V_m$. Choosing for example $l = 30$ cm and $V_m = 100$ V we find $N_0 = S \times 2\,\mu s$, and $\Delta t = 5$ ns. To first order, there is no extra time spreading due to the finite rise time of the pulse. The total collection time is $(\pi/2)\sqrt{m/k} = 157$ ns. Using a few hundred mCi's of ^{58}Co, source strengths [16] $S \approx 10^6\,\mathrm{s^{-1}}$ are now possible. A 30 cm accelerator should therefore produce pulses with more than one positron per pulse. It is also interesting that if the length l is reduced to a few cm we should be able to make ultrashort (10^{-9}–10^{-10} s) positron pulses which would be useful for measuring positron surface and bulk annihilation lifetimes.

We will now obtain a more precise estimate of the pulse shape. Given the equation motion $\ddot{z} + \omega^2 z = 0$ with the particular solution $z = z_0 \cos\omega t + (\dot{z}_0/\omega)\sin\omega t$, the solution of Boltzmann's equation for an initial velocity distribution $\exp(-mv_z^2/2w)(2\pi w/m)^{-1/2}$ and an

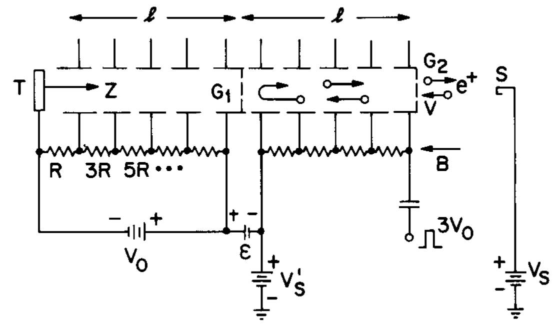

Fig. 2. Acceleration region for producing a simple harmonic oscillator (s.h.o.) potential $V = kz^2/2$. In the quiescent state, a steady positron flux from the positron moderator S enters the right half of the accelerator at low velocity $V \approx 3 \times 10^7\,\mathrm{cm\,s^{-1}}$ and is turned around by grid G_1. The left half of the accelerator has a potential which increases quadratically with z. This is accomplished by the applied voltage V_0 and the accelerator rings which are separated by appropriately chosen resistors, R, $3R$, $5R$, $7R$, ... This series of resistors continues into the right half of the accelerator where there is normally no applied voltage other than the potential $V_s' \approx V_s$ chosen to obtain the maximum possible density of positrons in this region. At $t=0$ a voltage step of magnitude $3V_0$ is applied to grid G_2. Immediately no more positrons can enter the accelerator, and those which were already in the accelerator are all destined to arrive at the larget T after a quarter s.h.o. period $2\pi\sqrt{m/k}/4$, where m is the positron mass

initial spatial distribution with density ϱ constant for $l<z<2l$ and zero elsewhere is

$$f(z, v_z, t) = \varrho \int_l^{2l} dz_0 \int_{-\infty}^{\infty} d\dot{z}_0 (2\pi w/m)^{-1/2} \exp(-m\dot{z}_0^2/2w) \cdot \delta(z - z_0 \sin\omega t - (\dot{z}_0/\omega)\cos\omega t) \cdot \delta(v_z - z_0\omega\cos\omega t + \dot{z}_0 \sin\omega t)\,. \quad (9)$$

The velocity distribution was chosen to be Maxwellian to simplify the integrations. The total flux at $z=0$ is

$$\phi(t) = \int f(0, v_z, t) v_z dv_z = [\varrho l/(2\pi)^{1/2}]\sigma t^{-2}[\exp(-t^2/2\sigma^2) - \exp(-4t^2/2\sigma^2)]\,, \quad (10)$$

where $\sigma^2 = (w/ml^2\omega^4)$ and we have assumed $\omega t \ll 1$. The peak flux at $t=0$ is $\phi(0) = 3\varrho l/2\sigma(2\pi)^{1/2}$. If we assume $\varrho = S(m/w)^{1/2}$ then $\phi(0) = [3/(2\pi)^{1/2}]S(V_0/w)$ where S is the source strength slow positrons per second, $4V_0$ is the total voltage applied to the accelerator in Fig. 2 and w is roughly the initial mean positron energy in the pulsed portion of the accelerator. The more precise expression in (10) is thus in agreement with the order of magnitude estimates used earlier. A plot of the expected distribution of positron arrival times represented by (10) is shown in Fig. 3.

In experiments which require the use of a low repetition rate laser ($\sim 10\,\mathrm{s^{-1}}$), the efficiency of the annihilation γ-ray detector may be small enough that we would like to have substantially more than one

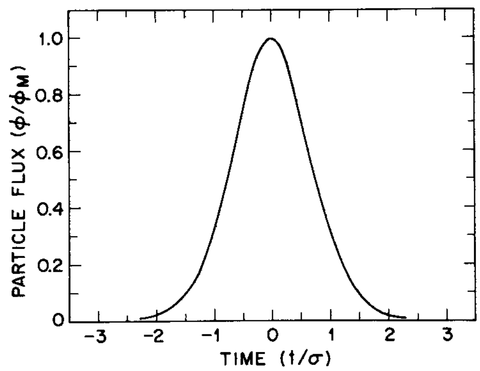

Fig. 3. Calculated pulse shape for the accelerator of Fig. 2. The unit of time is $\sigma=(w/2V_0)^{1/2}\omega^{-1}$ where w is the positron energy dispersion, $3V_0$ is the voltage pulse in Fig. 2 and ω is the harmonic oscillator frequency of a positron in the potential $V=kz^2/2$

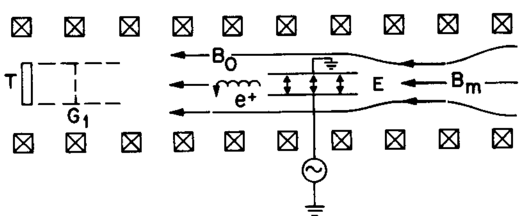

Fig. 4. Magnetic bottle rf trap to enhance the effectiveness of the bunching accelerator of Fig. 2. The mirror field is B_m. The rf transverse electric field E excites the positron cyclotron motion in the constant field B_0. The square objects represent the solenoid windings

positron in each pulse. As discussed above this will be possible if we sacrifice the good energy definition ($\varepsilon\approx 0.25$ eV) in the transverse directions as well as in the longitudinal direction. As shown in Fig. 4, we modify the experimental arrangement of Fig. 2 by adding a magnetic mirror field B_m and an rf cavity with a transverse electric field. The magnetic mirror transmits a charged particle if the ratio ($\varepsilon_\parallel/\varepsilon_\perp$) of longitudinal ($\varepsilon_\parallel$) to transverse ($\varepsilon_\perp$) energies is greater than B_m/B_0-1, where B_0 is the value of the magnetic field outside the mirror region. The rf cavity is driven at the positron cyclotron resonance frequency with an amplitude adjusted to impart a transverse energy equal to $\eta\varepsilon_\parallel$ the first time a positron enters the trap through the mirror field. Grid G_1 is adjusted to repel positrons of longitudinal energy $\varepsilon_\parallel$. If the magnetic field B_0 is uniform, positrons will return to the rf cavity in phase and will reflect off the magnetic mirror with $\varepsilon_\perp=2\eta\varepsilon_\parallel$. Because of the spread of longitudinal velocities the nonuniform mirror field will cause an indefinite phase shift. Upon next entering the mirror region, the transverse energy will be somewhere between 0 and $4\eta\varepsilon_\parallel$. After n reflections, $\varepsilon_\perp$ will have a roughly Gaussian distribution with a full width at half maximum $\Delta\varepsilon_\perp(n)\approx 2.35\sqrt{n-l}\,2\eta\varepsilon_\parallel$. Supposing that $B_m=2B_0$, the probability that a positron escapes from the trap on the n^{th} reflection is approximately $P_n=\varepsilon_\parallel/\Delta\varepsilon_\perp(n)\approx(4.7\sqrt{n-l}\,\eta)^{-1}$. Thus the probability that a positron does not survive n reflections is $\sum_{n'=2}^{n} P_{n'}\approx\sqrt{n-l}/(9\eta)$. The mean number of reflections is $\bar{n}\approx(9\eta)^2$ and the transverse energy spread will be $\sim 2\Delta\varepsilon_\perp(\bar{n})\approx 90\eta^2\varepsilon_\parallel$. Our design goal of keeping energy spreads under 100 eV is met if we choose $\eta\approx 2$ with $\varepsilon_\parallel=0.25$ eV. We will then have $\bar{n}\approx 400$ as expected from the simple arguments based on Liouville's theorem [18].

The gain in positron density implied by this value of $\bar{n}$ means that one should be able to produce ~ 10 ns long pulses containing 10^2–10^3 positrons. One concludes that the time bunching of a slow positron beam can be a very effective technique for studying time-dependent surface phenomena with positrons. It is interesting to note that further large gains in bunching efficiency can be obtained by exploiting dissipative forces which are exempt from the restrictions of Liouville's theorem. A time bunched beam can be moderated by a suitable high efficiency slow position emitting surface to obtain a monochromatic pulsed source which can be compressed further by a second accelerator stage if desired.

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